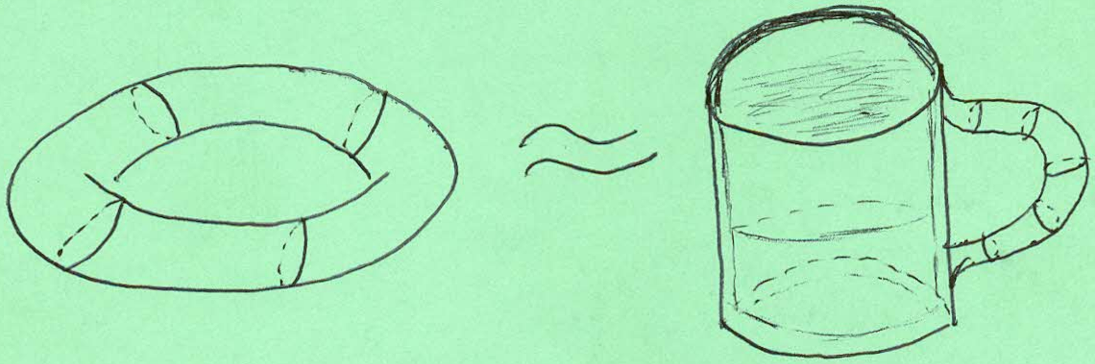
Topology

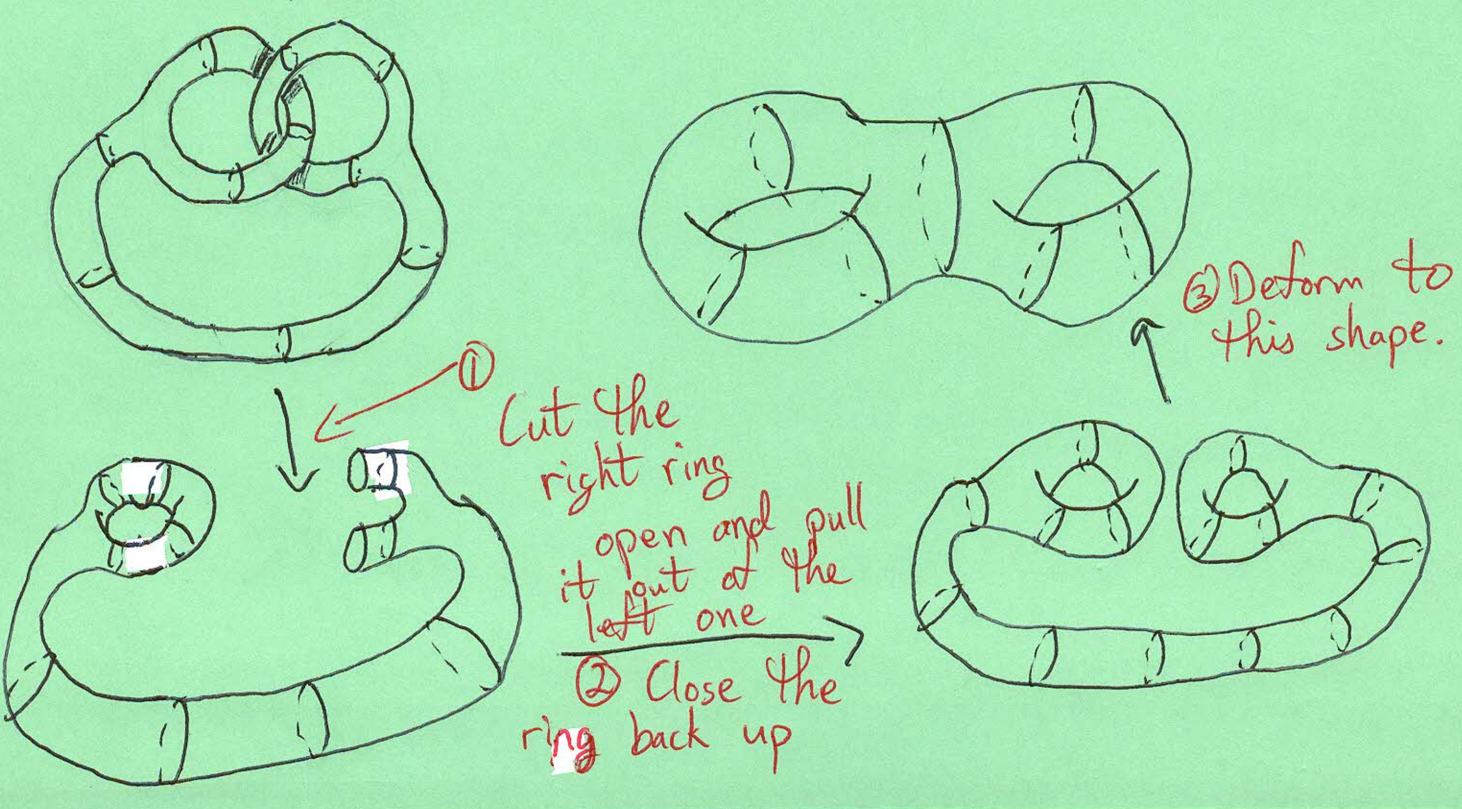
In topology, distances don't matter, so we are able to deform stuff as if it were made of clay. The quintessential example from topology is to show that the doughnut and the coffee mug are the same.



You might remember from highschool that two shapes/objects which were "the same" were called congruent. This required the sides to be the same length, angles to match, etc... however, in topology, those things are meaningless. In topology, two objects which are "the same" are called <u>homeomorphic</u>. How can we check for this?

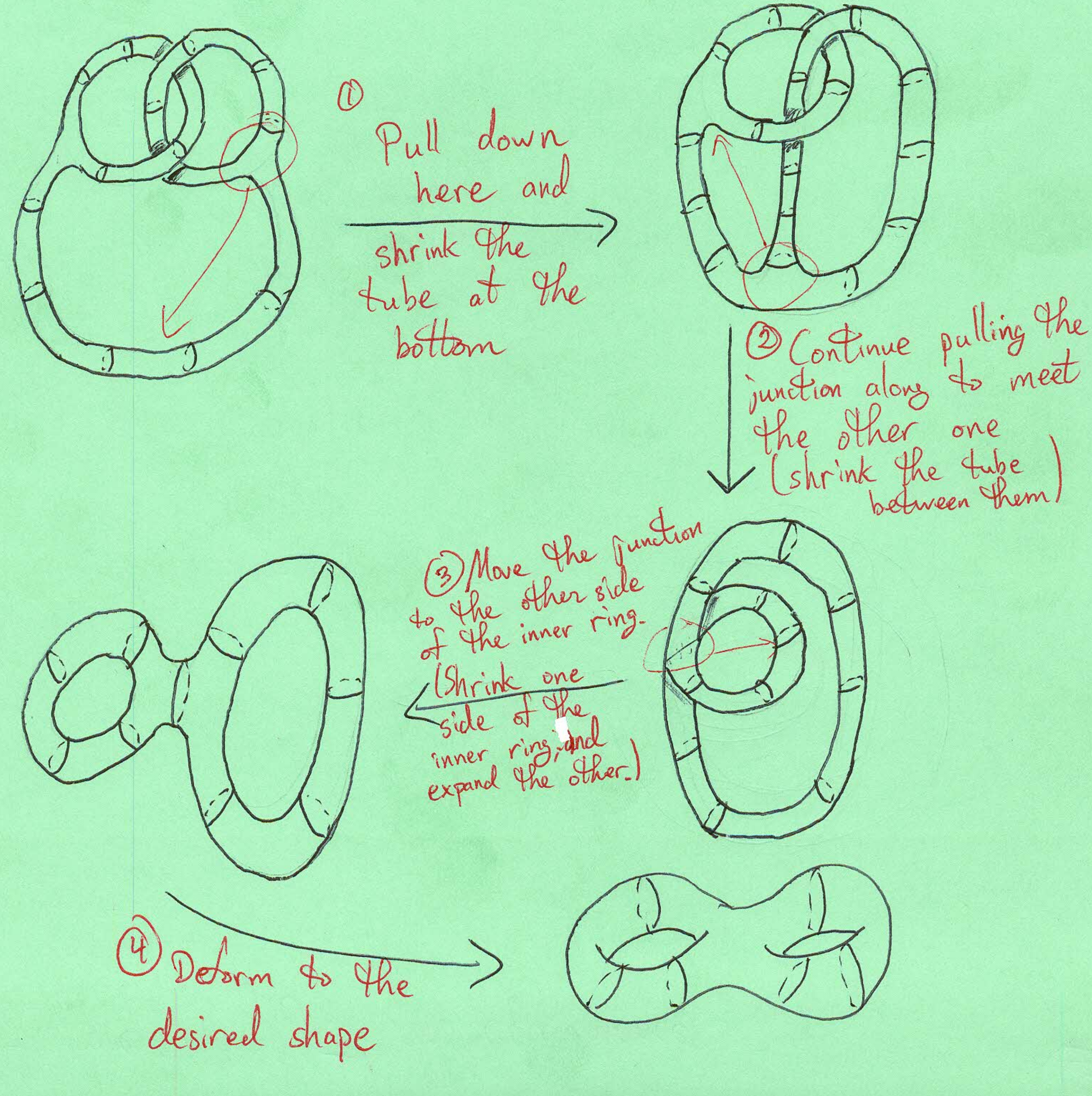
Since the surface is flexible, we can try to deform one shape to the other, as long as we don't make any new holes, pinch the shape (e.g ) or plug any holes. We're free to stretch/shrink/bend as we please. For homeomorphisms, we can also cut through the shape, as long as we put it back in the same way.

<u>Ex</u>: Show the following two surfaces are homeomorphic:



If we can deform one surface to another <u>without</u> cutting and reconnecting it, we say the surfaces are <u>ambiently isotopic</u>.

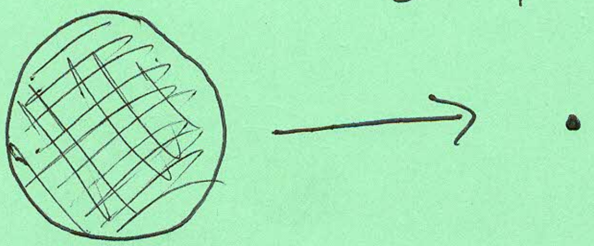
<u>Ex</u>: The two surfaces in the previous example are also ambiently isotopic.



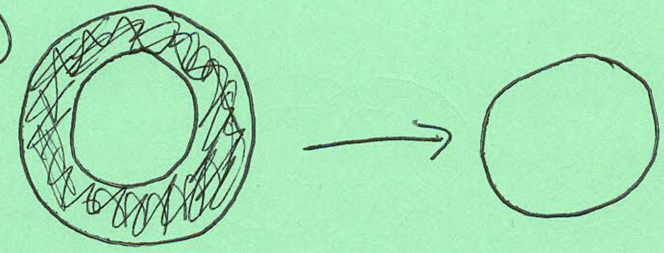
The last type of equivalence we'll cover is <u>homotopy equivalence</u>. This is like isotopy, but we can pinch stuff as thin as we like.

<u>Ex</u>: The following pairs are homotopy equivalent:

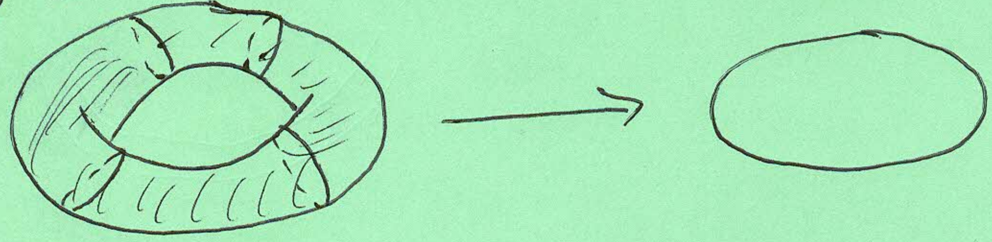
(a)



(b)

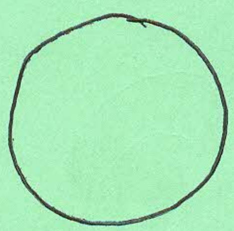


(c) (solid torus)

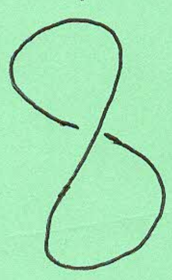


<u>Ex</u>: Which are homeomorphic? isotopic? homotopy equivalent?

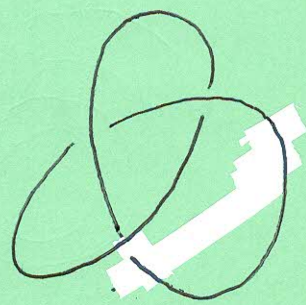
(a)



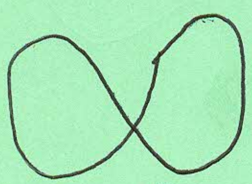
(b)



(c)

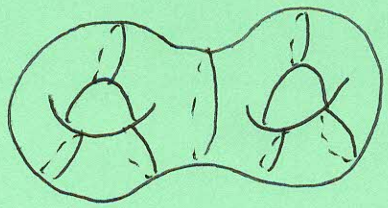


(d)

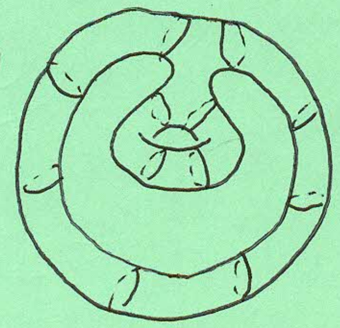


(Unlike (b), this one actually crosses itself)

(e)



(f)



Sol

Homeomorphic

$\boxed{1}$ a b c $\boxed{2}$ d $\boxed{3}$ e f

Isotopic

$\boxed{1}$ a b $\boxed{2}$ c $\boxed{3}$ d $\boxed{4}$ e f

Homotopic

$\boxed{1}$ a b $\boxed{2}$ c $\boxed{3}$ d e f